# Scalar Product

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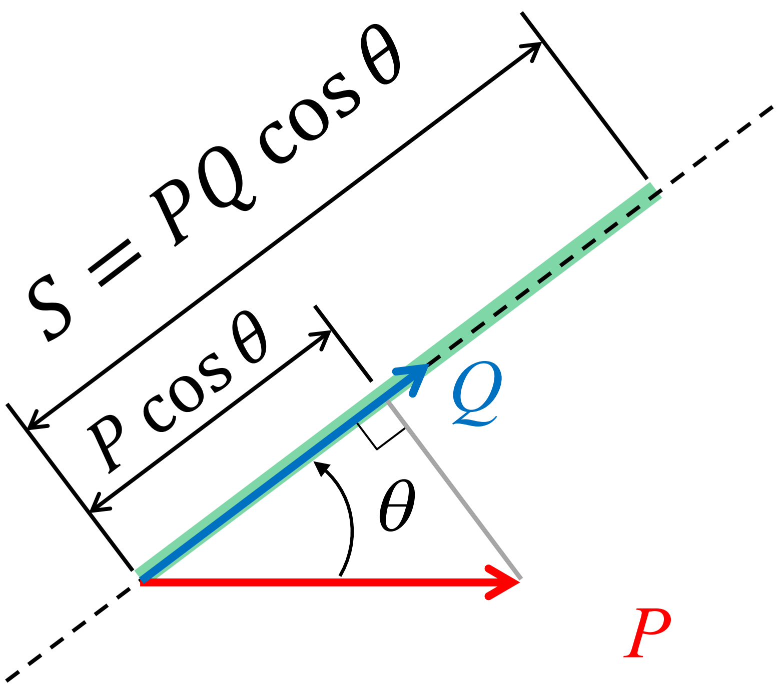
# Definition of the Scalar Product of Two Vectors

Consider two vectors in space and let $\theta$ be the angle between the two vectors

The scalar product is also commonly referred to as the Dot Product




$$S = \boldsymbol{P} \cdot \boldsymbol{Q}$$

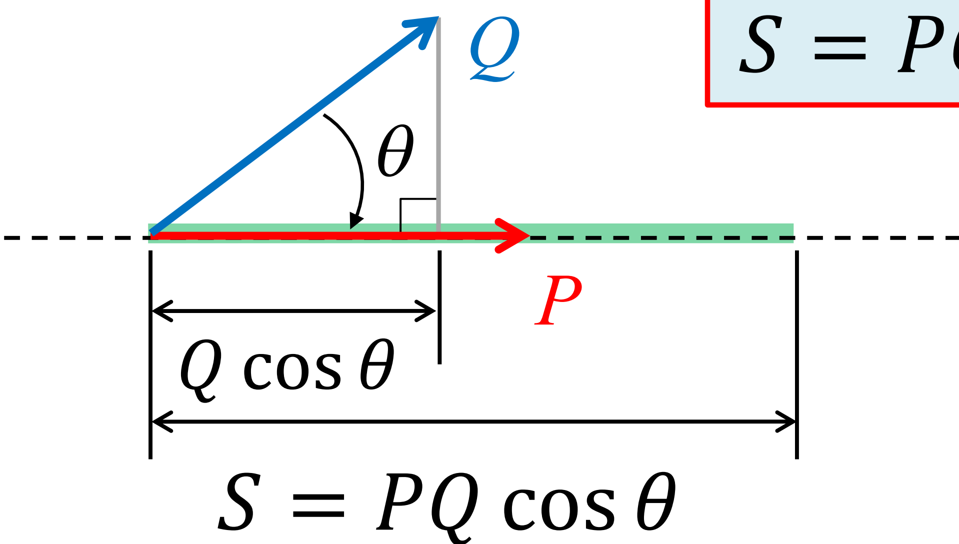
$$S = PQ\cos\theta$$

The scalar product can be thought of as the projection of vector $\boldsymbol{P}$ onto the line-of-action of vector $\boldsymbol{Q}$ multiplied by the magnitude of $\boldsymbol{Q}$

# Order of the Scalar Product Operation

$$S = \boldsymbol{P} \cdot \boldsymbol{Q} = \boldsymbol{Q} \cdot \boldsymbol{P}$$

$$S = PQ \cos\theta$$




The order of the operation does not change the scalar product

# Scalar Products of Unit Vectors

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = (1)(1)\cos(90°) = 0$$

$$S = PQ\cos\theta$$

Similarly;

$$\hat{\boldsymbol{\imath}} \cdot \widehat{\boldsymbol{k}} = 0$$

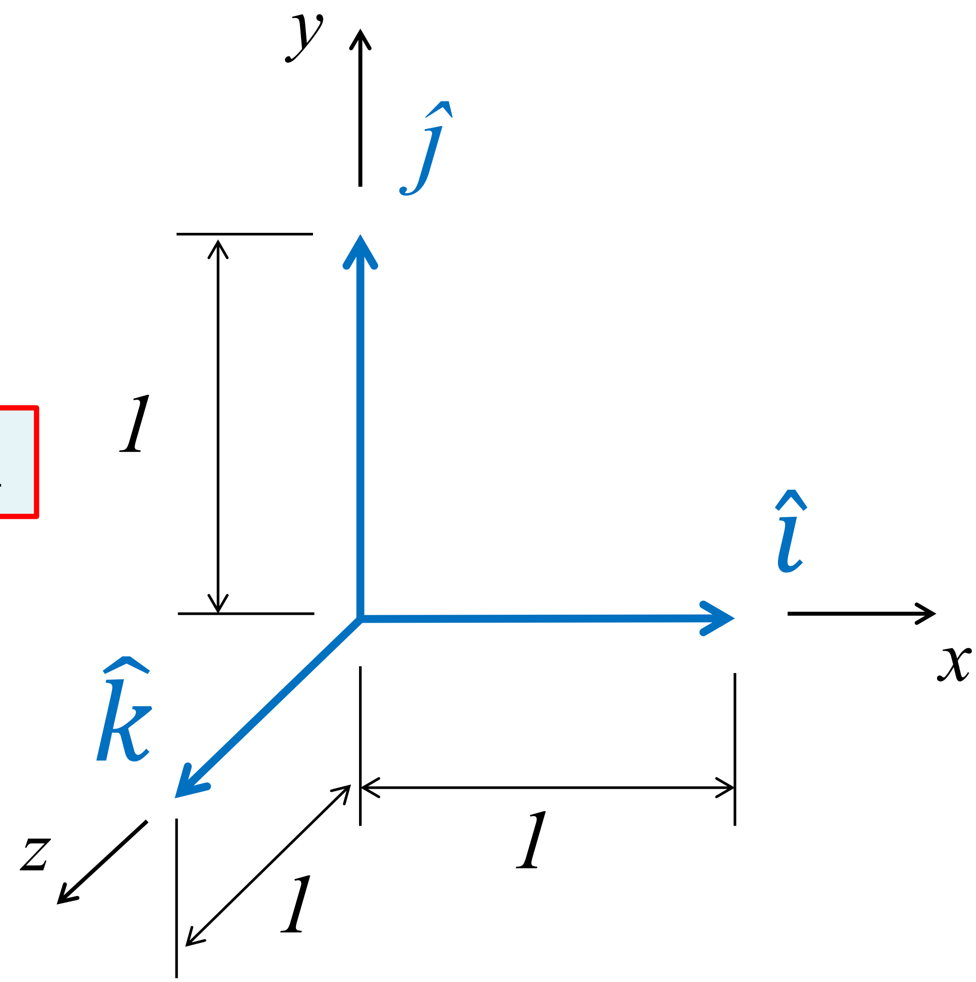
$$\hat{\boldsymbol{\jmath}} \cdot \widehat{\boldsymbol{k}} = 0$$

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = (1)(1)\cos(0°) = 1$$

Similarly;

$$\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = 1$$

$$\widehat{\boldsymbol{k}} \cdot \widehat{\boldsymbol{k}} = 1$$



# Scalar Product of Two Vectors in Cartesian Vector Form

$$S = \boldsymbol{P} \cdot \boldsymbol{Q}$$

***P*** and ***Q*** expressed in Cartesian Vector Form

$$\boldsymbol{P} = P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k}$$

$$\boldsymbol{Q} = Q_x\hat{\imath} + Q_y\hat{\jmath} + Q_z\hat{k}$$

$$S = \left(P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k}\right) \cdot \left(Q_x\hat{\imath} + Q_y\hat{\jmath} + Q_z\hat{k}\right)$$

$$S = P_xQ_x + P_yQ_y + P_zQ_z$$